Group Synchronization on Complex Networks: Kuramoto Model Meets Nonconvex Optimization

SPEAKER: Shuyang Ling, NYU Shanghai

TIME: 1:45 pm-2:45 pm, Tuesday, November 12, 2019

VENUE: Room 310, Pudong Campus, 1555 Century Avenue
(上海纽约大学310教室, 上海市浦东新区世纪大道1555号)

ABSTRACT

Studying the landscape of nonconvex cost function is key towards a better understanding of optimization algorithms in signal processing, statistics, and machine learning. Meanwhile, the famous Kuramoto model has been an important mathematical model to study the synchronization phenomena of coupled oscillators over network topologies. We bring together these two seemingly unrelated objects by investigating the optimization landscape of a nonlinear function $E(\theta) = \frac{1}{2}\sum_{i,j} a_{ij}(1-\cos(\theta_i-\theta_j))$ associated to an underlying network and exploring the relationship between the existence of local minima and network topology. This function arises in Burer-Monteiro method applied to $\mathbb{Z}_2$-synchronization as well as XY model in spin glass. Moreover, it corresponds to the energy function of the homogeneous Kuramoto model on complex networks for coupled oscillators. We prove the minimizer of the energy function is unique up to a global translation under deterministic dense graphs and Erd\’os-Rényi random graphs. Consequently, the stable equilibrium of the corresponding homogeneous Kuramoto model is unique and the basin of attraction for the synchronous state of these coupled oscillators is the whole phase space minus a set of measure zero. This result also addresses when the Burer-Monteiro method recovers the ground truth exactly from highly incomplete observations in $\mathbb{Z}_2$-synchronization. In addition to that, several open problems (some related to random matrix) will be discussed.

BIOGRAPHY

Shuyang Ling is currently an Assistant Professor Faculty Fellow of Data Science at NYU Shanghai. Prior to his joining in NYU Shanghai, he was a Courant Instructor at Courant Institute of Mathematical Science and Center for Data Science at New York University. He got his Ph.D. in applied mathematics from University of California Davis in 2017. His research focuses on applied mathematics, optimization, probability theory, and applications in engineering and machine learning problems.