The Scaling Limit of a Critical Random Directed Graph

SPEAKER: Robin Stephenson, University of Oxford
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ABSTRACT

We consider the random directed graph $D(n,p)$ with vertex set \{1,2,...,n\} in which each of the $n(n-1)$ possible directed edges is present independently with probability $p$. We are interested in the strongly connected components of this directed graph. A phase transition for the emergence of a giant strongly connected component is known to occur at $p = 1/n$, with critical window $p = 1/n + \lambda n^{-4/3}$ for $\lambda \in \mathbb{R}$. We show that, within this critical window, the strongly connected components of $D(n,p)$, ranked in decreasing order of size and rescaled by $n^{-1/3}$, converge in distribution to a sequence $(C_1,C_2,\ldots)$ of finite strongly connected directed multigraphs with edge lengths which are either 3-regular or loops. The convergence occurs in the sense of an $L^1$ sequence metric for which two directed multigraphs are close if there are compatible isomorphisms between their vertex and edge sets which roughly preserve the edge lengths. Our proofs rely on a depth-first exploration of the graph which enables us to relate the strongly connected components to a particular spanning forest of the undirected Erdős-Rényi random graph $G(n,p)$, whose scaling limit is well understood. We show that the limiting sequence $(C_1,C_2,\ldots)$ contains only finitely many components which are not loops. If we ignore the edge lengths, any fixed finite sequence of 3-regular strongly connected directed multigraphs occurs with positive probability.

BIOGRAPHY

Robin Stephenson obtained his PhD at Université Paris-Dauphine, and has been a post-doctoral researcher at the University of Zürich, NYU Shanghai and the University of Oxford. He recently was appointed assistant professor in applied probability at the University of Sheffield. His research interests are centered around random graphs and trees, and their scaling and local limits.