On the Liouville Type Theorems for the Stationary Navier-Stokes Equations

SPEAKER: Dongho Chae, Chung-Ang University
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ABSTRACT

We consider the stationary Navier-Stokes equations in $\mathbb{R}^3$

$$-\Delta u + (u \cdot \nabla) u = -\nabla p, \quad (1)$$

$$\nabla \cdot u = 0. \quad (2)$$

The standard boundary condition to impose at the spatial infinity is

$$u(x) \rightarrow 0 \quad \text{as} \quad |x| \rightarrow 0. \quad (3)$$

We also assume the finiteness of the Dirichlet integral,

$$\int_{\mathbb{R}^3} |\nabla u|^2 \, dx < +\infty. \quad (4)$$

Obviously $(u, p)$ with $u = 0$ and $p =$constant is a trivial solution to (1)-(4). A very challenging open question is if there is another nontrivial solution. This Liouville type problem is wide open, and has been actively studied recently in the community of mathematical fluid mechanics. The explicit statement of the problem is written in Galdi’s book [1, Remark X. 9.4, pp. 729], where under the stronger assumption $u \in L^2(\mathbb{R}^3)$ he concludes $u = 0$. After that many authors deduce sufficient conditions stronger than (3) and/or (4) to obtain the Liouville type result. In this talk we review various previous results, and present recent progress in getting sufficient condition in terms of the potential functions of the velocity.

BIOGRAPHY

Dongho Chae is currently CAU Distinguished Professor in Chung-Ang University in Korea. He obtained his Ph.D. in Princeton University, supervised by Professor Andrew Majda. His research interest lies in the theoretical studies of nonlinear partial differential equations arising in mathematical physics.